Un' urna A contiene 7 palline bianche e 3 p. nere.

Si estraggono 5 p. senza rimpiazzo.

Sia $X = n^\circ$ di p. bianche estratte.

Successivamente nell'urna B, inizialmente vuota, si mettono le X palline bianche e $5-X$ palline nere

Infine da B si esegue 1 estraz.

(a) Calcolare densità e media di X

(b) Calcolare la prob. di estrarre da B una p. bianca, sapendo che $X = 3$

(c) Calcolare la prob. di estrarre da B una p. bianca

ott 31-13:57

$X =$ n° di p. bianche estratte

$$P(X=k) = \frac{\binom{7}{k}\binom{3}{5-k}}{\binom{10}{5}} \quad \text{per } k = 2,3,4,5$$

$P(X=k) = 0$ altrimenti

$$\begin{cases} \boxed{0 \le k \le 7} \\ 0 \le 5-k \le 3 \longrightarrow \boxed{2 \le k \le 5} \end{cases}$$

$$P(X=2) = \frac{\binom{7}{2}\binom{3}{3}}{\binom{10}{5}} = \frac{\cancel{7!}}{\cancel{2!}\,\cancel{5!}} \cdot \frac{\cancel{5!}\,\cancel{5!}}{\cancel{10!}} = \frac{1}{12}$$

$$\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5} \;\; 2 \qquad \cancel{8}\cdot\cancel{9}\cdot\cancel{10} \;\; 4 \;\; 3$$

$P(X=3) = \frac{5}{12}$; $P(X=4) = \frac{5}{12}$

$P(X=5) = \frac{1}{12}$

$$E[X] = \sum_{k=2}^{5} k \, P(X=k) =$$

$$= 2 \cdot \frac{1}{12} + 3 \cdot \frac{5}{12} + 4 \cdot \frac{5}{12} + 5 \cdot \frac{1}{12} =$$

$$= \frac{42}{12} = \frac{7}{2} = 3.5$$

$$Z_i = \begin{cases} 1 & \text{se esce p. bianca all'i-esima estraz.} \\ 0 & \text{------ se no} \end{cases}$$

$$i = 1, \dots, 5$$

$$X = \sum_{i=1}^{5} Z_i$$

$$5 \cdot \frac{7}{10} = \frac{7}{2}$$

$$\rightarrow E[X] = \sum_{i=1}^{5} \underbrace{E[Z_i]}_{=\frac{7}{10}} = \sum_{i=1}^{5} \left(\frac{7}{10}\right)$$

$$E[Z_1] = \frac{7}{10};$$

$$Z_i \sim B(1, p_i)$$

$$E[Z_2] = \frac{7}{10}$$

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$$X \sim B(n, p)$$

$$\operatorname{Var} X = n p (1-p)$$

$$\cancel{X} = \sum_{i=1}^{n} Z_i$$

$$\operatorname{Var} X = \sum_{i=1}^{n} \underbrace{\operatorname{Var} Z_i}_{p(1-p)} = n p(1-p)$$

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$$X = \sum_{i=1}^{n} Z_i \leftarrow$$

$$\boxed{\operatorname{Var} X = \sum_{i=1}^{n} \underline{\operatorname{Var} Z_i} + \cancel{\sum_{i \neq j} \operatorname{cov}(Z_i, Z_j)}}$$

$$X = \sum_{i=1}^{5} Z_i \qquad Z_i \sim B\left(1, \frac{7}{10}\right)$$

$$\operatorname{cov}(Z_1, Z_2) = ?$$

$$= \underbrace{E[Z_1 \cdot Z_2]}_{?} - \underbrace{E[Z_1]}_{\frac{7}{10}} \cdot \underbrace{E[Z_2]}_{\frac{7}{10}} =$$

$$\underline{Z_1} \cdot \underline{Z_2} = \begin{cases} 1 & \text{se } Z_1 = 1 \text{ e } Z_2 = 1 \\ 0 & \end{cases} \sim B(1, p)$$

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(b) $P(\text{bianca} \mid \underline{X=3}) = \dfrac{3}{5}$

5 palline

3 bianche 2 nere

(c) $P(\text{bianca}) =$

$\{A_i\}$ partizione di Ω

B

$$P(B) = \sum_i P(B \mid A_i)\, P(A_i)$$

ott 31-14:57

$$A_i = \{X=i\} \qquad i=2,3,4,5$$

$$P(\text{bianca}) = \sum_{i=2}^{5} P(\text{bianca} \mid X=i) \cdot P(X=i)$$

$$P(\text{bianca} \mid X=i) = \frac{i}{5}$$

$$P(X=i) = \frac{\binom{7}{i}\binom{3}{5-i}}{\binom{10}{5}}$$

$$P(\text{bianca}) = \sum_{i=2}^{5} \frac{i}{5} \cdot \underbrace{\frac{\binom{7}{i}\binom{3}{5-i}}{\binom{10}{5}}}_{P(X=i)} = \dots$$

$$= \frac{1}{5} \boxed{\sum_{i=2}^{5} i \, P(X=i)} = \frac{1}{5} E[X] = \frac{7}{10}$$

$$p = P(Z_1 = 1, Z_2 = 1) = P(Z_2 = 1 \mid Z_1 = 1)\, P(Z_1 = 1)$$

$$= \frac{6}{9} \cdot \frac{7}{10}$$

$$= \frac{42}{90}$$

$$\mathrm{cov}(Z_1, Z_2) = \frac{7}{15} - \left(\frac{7}{10}\right)^2 = \dots$$

$$\mathrm{cov}(Z_2, Z_3) =$$

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p $\quad i = 1, 2, 3 \ldots$

$$X_i = \begin{cases} 1 & \text{se l'i-esimo cliente è sottoposto a rilettura} \\ 0 & \text{se no} \end{cases}$$

(a) Calcolare le legge di X_i, $\forall i$

(b) 10 clienti in totale.

Z = n° di clienti sottoposti a rilettura

Calc. la legge di Z

(c) Supponiamo che si presentino N clienti, dove N è una v. a. avente legge Π_λ. Calcolare

Poisson di par λ

$$P(Z = 3 / N = 20)$$

e

$$P(Z = 3 , N = 20)$$

$\cap$

ott 31-15:04

(a) $X_i \sim B(1, p)$ $\forall i$
(indipendenti)

(b) 10 clienti (10 prove)

$Z =$ n° di successi in 10 prove bernoulliane ripetute indipend. di par p

$Z \sim B(10, p)$

(c) $P(Z = 3 \mid N = 20) =$

$$\binom{20}{3} p^3 (1-p)^{20-3}$$

$$P(Z = \overset{h}{3}, N = \overset{k}{20}) = P(Z = 3 \mid N = 20)\, P(N = 20)$$

$$= \binom{20}{3} p^3 (1-p)^{20-3} \cdot \frac{\lambda^{20}}{20!} e^{-\lambda}$$

(d) Calc. la legge di Z.

$P(Z = h, N = k) =$

$$\begin{cases} \binom{k}{h} p^h (1-p)^{k-h} \dfrac{\lambda^k}{k!} e^{-\lambda} & \boxed{h = 0, 1, \dots, k\ ;\ k = 0, 1, 2, 3, \dots} \\ 0 & \end{cases}$$

$$P(Z = h) = \sum_k P(Z = h, N = k) =$$

$$= \sum_{k=h}^{\infty} \binom{k}{h} p^h (1-p)^{k-h} \frac{\lambda^k}{k!} e^{-\lambda}$$

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$$\sum_{k=h}^{\infty} \binom{k}{h} p^h (1-p)^{k-h} \frac{\lambda^k}{k!} e^{-\lambda} =$$

$$= \sum_{k=h}^{\infty} \frac{\cancel{k!}}{h!\,(k-h)!}\, p^h (1-p)^{k-h} \frac{\lambda^h \lambda^{k-h}}{\cancel{k!}} e^{-\lambda}$$

$$= \sum_{k-h=0}^{\infty} \underbrace{\frac{(p\lambda)^h}{h!}} \frac{[(1-p)\lambda]^{k-h}}{(k-h)!} \underbrace{e^{-\lambda}}$$

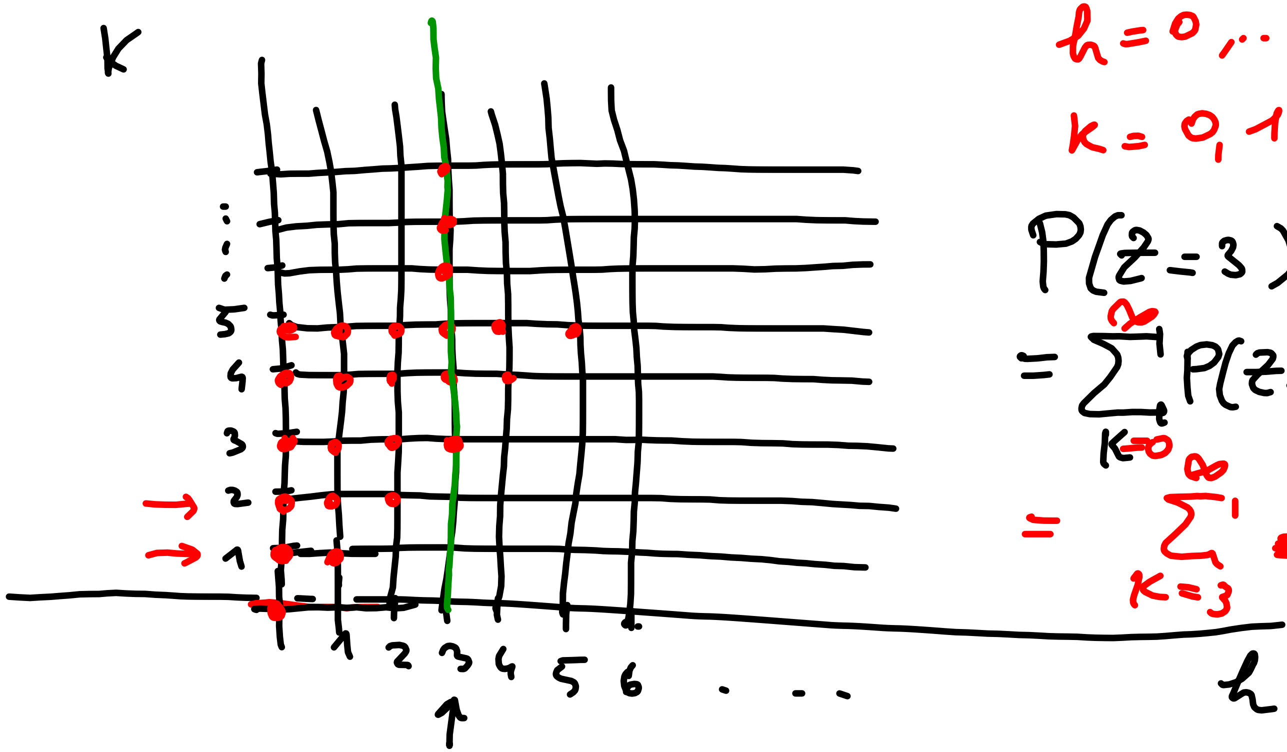
$k-h=n$

$$= \frac{(p\lambda)^h}{h!} e^{-\lambda} \left\{ \sum_{n=0}^{\infty} \frac{[\overbrace{(1-p)\lambda}^{x}]^n}{n!} \right\} =$$

$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x} = \frac{(p\lambda)^h}{h!} e^{-\lambda} e^{(1-p)\lambda}$$

$$= \frac{(p\lambda)^h}{h!} e^{-(p\lambda)} = P(Z=h)$$

$\lambda \quad p=\frac{1}{2} \qquad Z \sim \Pi_{(p\lambda)}$



$h = 0, \ldots\ k$

$k = 0, 1 \ldots$

$$P(Z=3)$$

$$= \sum_{k=0}^{\infty} P(Z=3, N=k)$$

$$= \sum_{k=3}^{\infty}{}'$$

$$P(Z_2=1) = P(Z_2=1, Z_1=1) + P(Z_2=1, Z_1=0) =$$

$$\{Z_2=1\} = \{Z_2=1, Z_1=1\} \cup \{Z_2=1, Z_1=0\}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$= P(Z_2=1|Z_1=1)\cdot P(Z_1=1) + P(Z_2=1|Z_1=0)\,P(Z_1=0)$$

$$= \frac{6}{9}\cdot\frac{7}{10} + \frac{7}{9}\cdot\frac{3}{10}$$

$$= \frac{7}{10\cdot 9}(6+3) = \frac{7}{10}$$

ott 31-14:42